SOME EXAMPLES OF THE APPLICATION OF INFORMATION TECHNOLOGY IN MATHEMATICS

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1. ABSTRACT

Creating an information society both in Europe and in Lithuania is speeding up. Today's training is no longer imaginable without information technology. The society is more and more computerized. Information technology is increasingly integrated into the teaching and learning process, becoming an integral part of it. Up-to-date information tools and technologies are encouraged to be used at all levels of the educational process, teaching and learning various subjects. Information technology has a tremendous impact on teaching and learning, making it more attractive and more visible.

Mathematics is the main foundational discipline not only for science and technology, but also for various other spheres of human activity.

Different types of change, which vary in quality and in difficulty present different challenges to the innovators. Process changes in higher mathematics education oriented around mathematics pure and applied.

The main objective in studying mathematics using computer is to have students actually doing mathematics, rather than improving student's information processing abilities. Studying mathematics with computers it be can easy for students, because usually they get good informatics technology knowledge in school.

2. FIELDS OF APPLICATION OF INFORMATION TECHNOLOGIES IN MATHEMATICAL TRAINING

That is why the teaching staff must manage different teaching systems. The broader the range of teaching methods, the better they can be applied to different situations that appear during the study process. New methods and tendencies appear also in teaching mathematics. Usage of computers and mathematical programs are spreading more and more for mathematical calculations, as well as for modelling mechanic and economic processes. [3]

Teaching mathematical programs parallel to a traditional course is being introduced all over the world, MATHCAD and MATLAB are the most popular programs for that [3]. Mathcad has a simpler interface [2].

Computer simulation in mathematics can be especially useful. We support the use of computer for the mathematical learning of students.

Using programs Mathcad and Microsoft Excel we demonstrate mathematics courses:

- vectors, operations on vectors, matrix;
- complex numbers and solutions of equations;
- solving systems of linear equation;
- functions and limits;
- differential calculus and solving differential equations;
- integral calculus: indefinite and definite integrals, applications of integration;
- 3D Scatter plot of a function;
- solving and optimization extension;
- calculation of probability;
- statistics: arrangement of data, correlation, analysis of data and hypothesis,
- Logic, Bull Functions.

What students understand mainly in mathematics is applied mathematics. They can use computers and solve many problems. Programs MATHCAD and MATLAB can solve many mathematics problems including programming: technical calculating, operations research and manufacturing and other technical and business fields.

Students acquire mathematical programs with interest. Work with these programs helps students understand the algorithm of the task. Students using mathematical programs can solve tasks in a short time. Using information technology possibilities in teaching mathematics eases pedagogues work and also makes the learning process more interesting and effective. [3]

Using the information technology possibilities in teaching mathematics eases the pedagogues' work, and also makes the learning process more interesting and effective. It also significantly improves the quality of education and the level of preparedness in the labour market. [3]

3. SPIN ON THE OX AXIS, VOLUME AND SURFACE AREA CALCULATION

The visualization of 3D world, this term is used for viewing real-world objects such as human body, buildings and the selection of all received information. This form of visualization should be applied when looking at an object in a 3D space.

Most engineering students have unimaginable imagination when it comes to solving the tasks. In order to solve these problems, we must help students and develop their geometric imagination, therefore, information technology are integrated into mathematical training.

We will present an example of how to use information technology in teaching of spins. To train spins together with addressing the functions of creation and spin tasks.

Task: *With Lagrange interpolating polynomial, calculate the curvilinear trapezoid area and perimeter. Construct a spin by turning the curvilinear trapezoid on the Ox axis at an angle of 360°. Calculate the resultant spin surface area and volume. (Fig. 1)*

Solving this task made Lagrange interpolating polynomial. Using the table of values one can create the Lagrange interpolating polynomial. It is drawn up by the program MATHCAD. (Fig. 2)

Students are taught how to calculate curvilinear trapezoid area and perimeter. Table 1 shows the formulas used and the computing results.
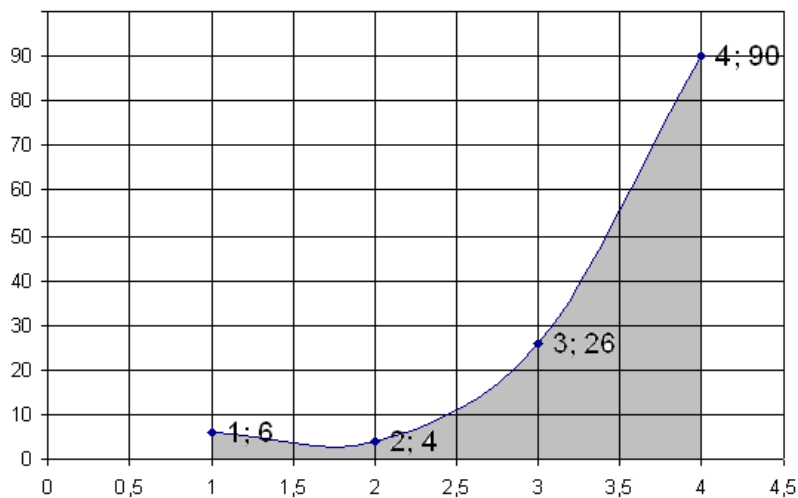

Figure 1.
Initial data

$$L(x) := \frac{(x-2)\cdot(x-3)\cdot(x-4)}{(1-2)\cdot(1-3)\cdot(1-4)}\cdot 6 + \frac{(x-1)\cdot(x-3)\cdot(x-4)}{(2-1)\cdot(2-3)\cdot(2-4)}\cdot 4 + \frac{(x-1)\cdot(x-2)\cdot(x-4)}{(3-1)\cdot(3-2)\cdot(3-4)}\cdot 26 + \frac{(x-1)\cdot(x-2)\cdot(x-3)}{(4-1)(4-2)(4-3)}\cdot 90$$

$$L(1) = 6 \qquad L(2) = 4 \qquad L(3) = 26 \qquad L(4) = 90$$

$$L(x) \text{ simplify} \rightarrow 3\cdot x^3 - 6\cdot x^2 - 5\cdot x + 14$$

$$f(x) := 3\cdot x^3 - 6\cdot x^2 - 5\cdot x + 14$$

Figure 2.
Lagrange interpolating polynomial of the function given in the table

Formula	Result calculation in program Mathcad
A curvilinear trapezium bounded FUNCTIONS f(x)≥0 graph, the abscissa axis (Ox) and the straight lines x = a, x = b area $$S = \int_a^b f(x)dx$$	$$S := \int_1^4 f(x)\,dx$$ $$S = 69.75$$
Curve arc length $$\int_a^b \sqrt{1+y'^2}\,dx$$	$$\frac{d}{dx}f(x) \rightarrow 9{\cdot}x^2 - 12{\cdot}x - 5$$ $$l := \int_1^4 \sqrt{1 + \left(9{\cdot}x^2 - 12{\cdot}x - 5\right)^2}\,dx$$ $$l = 90.45$$ $$P := 6 + 3 + 90 + 90.45$$ $$P = 189.45$$ P – perimeter

Table 1.
Curvilinear trapezoid area and perimeter calculation

Formula	Result calculation in program Mathcad
The spin surface area $$2\pi\int_a^b y\sqrt{1+(y')^2}\,dx$$	$$Spav := 2{\cdot}\pi{\cdot}\int_1^4 f(x){\cdot}\sqrt{\left(9{\cdot}x^2 - 12{\cdot}x - 5\right)^2}\,dx$$ $$Spav = 25507.54$$
Spin volumetric calculate according to the formula: $$V = \pi\int_a^b y^2\,dx$$	$$V := \pi{\cdot}\int_1^4 (f(x))^2\,dx$$ $$V = 10795.95$$

Table 2.
The spin of the surface area and volume calculation

Turning the shape around the Ox axis is described by the function:

$$F(u, v) := u$$

$$G(u, v) := f(u) \cdot \cos(v)$$

$$H(u, v) := f(u) \cdot \sin(v)$$

$$S := \text{CreateMesh}(F, G, H, 1, 4, 0, 2 \cdot \pi, 30)$$

Figure 3.
Spin description

With Mathcad the depicted surface is obtained by the function of twisting in Ox axis. (Fig. 4)

The display of the spin fostered students' visual thinking. They can see the spin, rotate it at different angles. Using information technology spin training becomes images.

When calculating the spin characteristics one can visually assess the obtained values. (Table 2)

Studying spins the use of information technology; most students are capable enough to solve complex tasks. We move student's imagination and the capacity of applied mathematics knowledge into practice. Using program Mathcad less time is spent in technical computing. Spins are presented visually, developing students' visual thinking.

4. DIFFERENTIAL EQUATIONS NUMERICAL METHODS

Differential equations are one of the most applied mathematics disciplines to other areas of science. These equations, numerical solution methods improved application possibilities. Many differential equations modelled tasks are relatively complex. Differential equations, numerical solution methods for finding approximate solution of the task. To acquaint students with the numerical solution of differential equations methods Panevėžys University of Applied Sciences College of Engineering disciplines are taught 4 methods: Euler, Euler corrected, Euler modify, the fourth consecutive Runge Kutta method.
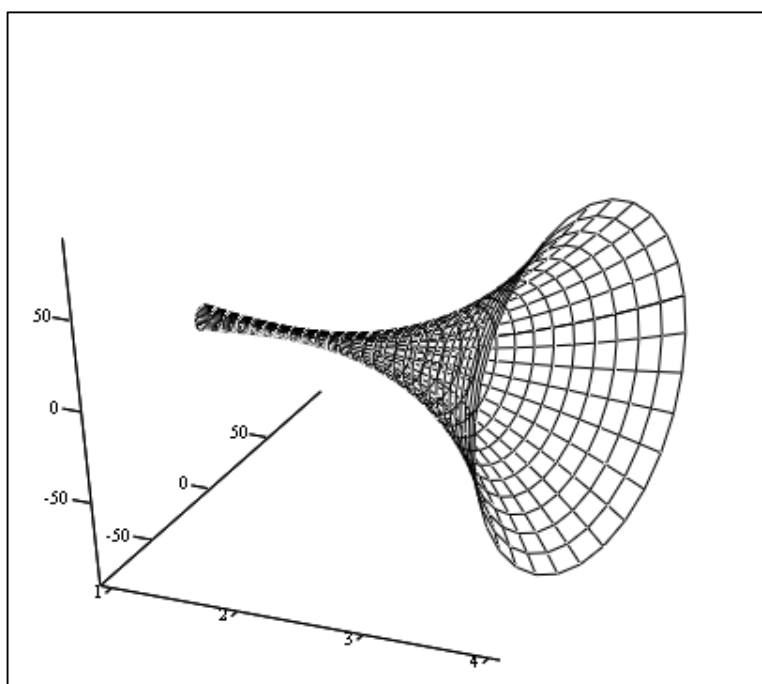
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Figure 4.
Spin obtained by rotating a function f(x) on the axis Ox

Task: *The equation $y' = f(x, y)$. You need to find its independent solution that satisfies the initial condition $y(x_0) = y_0$. Geometrically Euler method means that to search for the solution – an integral curve $y(x)$ replace Euler polyline.*

x varies step h:

$x_{k+1}=x_k+h$

Euler method is described:

$y_{k+1}=y_k+h \cdot f(x_k, y_k)$

Euler corrected method is described:

$y_{k+1}=y_k+1/2 \cdot h \cdot (f(x_k, y_k)+ f(x_k+h, y_k+h \cdot f(x_k, y_k)))$

Euler modify method is described:

$y_{k+1}=y_k+h \cdot f(x_k+1/2 \cdot h, y_k+1/2 \cdot h \cdot f(x_k, y_k)))$

In Applied mathematics course, in order to facilitate the calculations we use Excel.

Task: *With Euler method using Excel solve differential equation* $y' = \dfrac{3y}{x+2}$, $y(1)=3$; *create 10 values of h = 0.01.*

	A	B	C	D
1	Nr.	x_i	y_i	$f(x_i,y_i)$
2	0	1	3	=3*C2/(B2+2)
3	1	1,01	=C2+0,01*D2	=3*C3/(B3+2)
4	2	1,02	=C3+0,01*D3	=3*C4/(B4+2)
5	3	1,03	=C4+0,01*D4	=3*C5/(B5+2)
6	4	1,04	=C5+0,01*D5	=3*C6/(B6+2)
7	5	1,05	=C6+0,01*D6	=3*C7/(B7+2)
8	6	1,06	=C7+0,01*D7	=3*C8/(B8+2)
9	7	1,07	=C8+0,01*D8	=3*C9/(B9+2)
10	8	1,08	=C9+0,01*D9	=3*C10/(B10+2)
11	9	1,09	=C10+0,01*D10	=3*C11/(B11+2)
12	10	1,1	=C11+0,01*D11	=3*C12/(B12+2)

Figure 5.
Differential equation solution with Euler method

In the C column are written method formula, D – given function. Cell B2 – the initial value of x, C2 – the initial value of y. In the column C is the method formula, D – given function, E – function $(x_k+h, y_k+h \cdot f(x_k, y_k))$. In cell B2 – the initial value of x, C2 – the initial value of y. The exact solution is found using the program Matlab or Wolfram Alpha.

The error in Euler method is proportional to h^2. Euler's method is often unstable, so in practice this method is rarely used. For Euler revised (improved) and methods of modified restriction the error is proportional to h^3. These methods are second order Runge Kutta methods.

The analysis of errors is undeniable. We solve the differential equation in three methods. Differential equations selected such that it is resolved in analytical way too. This solves the differential equation by program Matlab. Therefore, we can compare calculation errors by different methods.

Task: *In Euler, Euler corrected and Euler modify methods using Excel to solve differential equation* $y' = \dfrac{3y}{x+2}$, $y(1)=3$ *creating 10 values of h=0.01. Compare the absolute and relative errors.*

The exact solution is found using the program Matlab. (Fig. 6)

To compare the errors information technology can be used. In Figure 7 for the differential equation 3 Euler type solution methods can be seen. The displayed numbers are obtained in MATLAB.

Comparing the solutions one can accurately count the absolute and relative errors. In Table 3 the calculated absolute errors are displayed.

```
>> dsolve('Dy=3*y/(t+2)','y(1)=3')

ans =

1/9*t^3+2/3*t^2+4/3*t+8/9
```

Figure 6.
Differential equation solution in program Matlab

	A	B	C	D	E	F	G	H	I
1	Nr.	x_i	y_i (Euler method)	$f(x_i,y_i)$	$f(xi+h,yi+h*f(xi,yi))$	y_i (Euler corrected method)	$f(xi+1/2*h,yi+1/2*h*f(xi,yi))$	y_i (Euler modify method)	The Exact solution
2	0	1	3	3	3,019933555	3	3,009983361	3	3
3	1	1,01	3,03000000	3,01993355	3,04003311	3,03009967	3,03004975	3,03009983	3,03010011
4	2	1,02	3,06019934	3,03993311	3,06019933	3,06040000	3,05018280	3,06040033	3,06040089
5	3	1,03	3,09059867	3,05999868	3,08043221	3,09090166	3,07038253	3,09090216	3,09090300
6	4	1,04	3,12119865	3,08013025	3,10073176	3,12160531	3,09064891	3,12160598	3,12160711
7	5	1,05	3,15199996	3,10032783	3,12109798	3,15251163	3,11098196	3,15251247	3,15251389
8	6	1,06	3,18300323	3,12059141	3,14153086	3,18362127	3,13138168	3,18362229	3,18362400
9	7	1,07	3,21420915	3,14092099	3,16203041	3,21493491	3,15184807	3,21493611	3,21493811
10	8	1,08	3,24561836	3,16131658	3,18259662	3,24645322	3,17238111	3,24645459	3,24645689
11	9	1,09	3,27723152	3,18177818	3,20322949	3,27817685	3,19298083	3,27817840	3,27818100
12	10	1,1	3,30904931	3,20230578	3,22392903	3,31010648	3,21364721	3,31010821	3,31011111

Figure 7.

For differential equation $y' = \dfrac{3y}{x+2}$, $y(1) = 3$ the solution in Euler,

Euler corrected and Euler modify methods can be seen. (h=0.01)

	Euler method	Euler corrected method	Euler modify method
0	0,00000000	0,00000000	0,00000000
1	0,00010011	0,00000044	0,00000028
2	0,00020155	0,00000089	0,00000056
3	0,00030433	0,00000134	0,00000084
4	0,00040846	0,00000180	0,00000113
5	0,00051393	0,00000226	0,00000142
6	0,00062077	0,00000273	0,00000171
7	0,00072896	0,00000320	0,00000200
8	0,00083853	0,00000367	0,00000230
9	0,00094948	0,00000415	0,00000260
10	0,00106181	0,00000463	0,00000290

Table 3.
Absolute errors

Euler method	Euler corrected method	Euler modify method
0,000000%	0,000000%	0,000000%
0,003304%	0,000015%	0,000009%
0,006586%	0,000029%	0,000018%
0,009847%	0,000043%	0,000027%
0,013087%	0,000058%	0,000036%
0,016305%	0,000072%	0,000045%
0,019503%	0,000086%	0,000054%
0,022679%	0,000099%	0,000062%
0,025836%	0,000113%	0,000071%
0,028972%	0,000127%	0,000079%
0,032088%	0,000140%	0,000088%

Table 4.
Relative errors

We assessed the relative error of the solution. The results are provided in percentage with 8 positions after the comma. Relative errors results shown in Table 4.

Selecting a larger step solution errors occur proportionally higher. Quickly compare the resulting errors can be the use of information technology. They not only speed up the decision, but also provides a graphical view.

To train cubic splines with the use of information technology develops mathematical thinking. Students have an opportunity to get a quick and visual image.

5. CONCLUSIONS

(A) After analysing the application of information technology in mathematics, we can state that we can apply them without abandoning the traditional methods of teaching mathematics.

(B) The method of calculating the surface area and volume of the spin using information technologies is presented, it can be used for studies.

(C) By studying the numerical methods of differential equations and using the Excel program, we can quickly obtain estimates and estimate errors.

6. REFERENCES

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