TEMPERATURE IN THE EARLY STAGE OF UNIVERSE

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1. ABSTRACT

A possible way is shown how to introduce naturally the concept of a dynamical temperature as a thermodynamic parameter in a cosmological model. This is done within the framework of classical field theory. Here, a scalar field is introduced that generates the dynamic temperature. Such a potential allows us to describe the time evolution of this fundamental thermal property in the framework of inflationary models of the early universe. In the model the inflation field decays into this additional field. The dynamics contains a phase transition dividing the energy propagation into a dissipative and a non-dissipative process involving a spontaneous symmetry breaking mechanism. The time evolution in the rapid expanding universe is described. The presented description is in line with cosmological inflationary models.

2. INTRODUCTION

The thermal behaviour of the rapidly change systems cannot be described by the static (local equilibrium) equations of the thermodynamics. For this reason the dynamic temperature definition has been introduced applying the Lorentz invariant mathematical frame within the Lagrangian formulation, finally, a relativistic invariant theory has been elaborated for the pure thermal transfer [1-6]. After this, the obvious aim was to connect this thermal field with other physical fields by which step the concepts of thermodynamics can appear in the theory in a natural way. In the present paper, an example is shown with the coupling of the cosmological inflaton field and this thermal field, by which a short period of the universe – the inflationary era – is treated [7-23]. Both of the two considered models, Linde's inflation model and dynamic temperature (DT) model, must have the same mathematical frame, now, this is the Lagrange-Hamiltonian theory. To build up the theory, first, the mathematical background of the cosmological description of the Linde-model is shortly summarized in Section 3. Then, the Lagrangian based construction of the dynamic temperature is shown in Section 4. The coupling of the interacting inflaton and thermal fields is treated in Section 5. The solution of the obtained equations of motion at an optional parameter set is discussed in Section 6.

3. EINSTEIN'S EQUATION AND THE EQUATIONS OF MOTION IN FIEDMAN-ROBERTSON-WALKER (FRW) METRIC

The action of a physical motion can be expressed by the Lagrangian L_{FRW} of the examined process and the metric of the space $\tilde{g}$

$$S = \int d^4x \sqrt{-\tilde{g}}\, L_{FRW} \,. \tag{1}$$

The Lagrangian of the Linde's inflation model can be formulated [7-23] by

$$L_{FRW} = \frac{1}{2}\left(\frac{\partial \phi}{\partial t}\right)^2 - \frac{1}{2a^2}(\nabla\phi)^2 - V(\phi)\,, \tag{2}$$

where ϕ is a scalar field, the so-called inflaton field, $a(t)$ pertains to the metric of the space; the normalized radius of the universe. In the present model, the metric $\tilde{g}$ is coming from the Friedman-Robertson-Walker (FRW) metric, and the determinant of this metric tensor:

$$\sqrt{-\tilde{g}} = a^3 . \tag{3}$$

Here, $a(t)$ is the normalized radius of the universe

$$a = \frac{R(t)}{R_0} . \tag{4}$$

The equation of motion of the inflaton field can calculated by the usual way which is

$$\frac{\partial^2 \phi}{\partial t^2} - \frac{1}{a^2}\Delta\phi + 3H\frac{\partial \phi}{\partial t} = -\frac{\delta V(\phi)}{\delta \phi} . \tag{5}$$

Here, the Hubble parameter H can be introduced as

$$H = \frac{\dot{a}}{a} = \frac{1}{a}\frac{\mathrm{d}a}{\mathrm{d}t}. \tag{6}$$

The different models depend on the choice of the potential of the interacting fields. The hybrid inflation model (Andrei Linde, 1993), which is the most accepted in the literature, is

$$V(\sigma,\phi) = \frac{1}{2}m^2\phi^2 + \frac{1}{2}g^2\phi^2\sigma^2 + \frac{1}{4\lambda}(M^2 - \lambda\sigma^2)^2, \tag{7}$$

where the first term pertains to inflaton field with the mass m; σ is the Higgs field, the second term couples the inflaton and the Higgs field; the third term generates Higgs mechanism where M is the mass of the Higgs boson (as we know from the LHC experiments M = 125.3 ± 0.6 GeV/c^2). The Hamiltonian (energy density) can be easily obtained

$$\tilde{H} = \rho_\phi = \left(\frac{1}{2}\left(\frac{\partial\phi}{\partial t}\right)^2 + \frac{1}{2a^2}(\nabla\phi)^2 + V(\phi)\right). \tag{8}$$

The calculations requires to express the Einstein equation in FRW metric

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho, \tag{9}$$

where ρ is the mass density, G is the gravitational constant. It is useful to introduce the Planck mass

$$M_{Pl} = \sqrt{\frac{\hbar c}{8\pi G}},$$

where $\hbar$ is the Planck constant per 2π. Then the Friedman equation – which corresponds to a flat universe (parabolic geometry, measured by the instruments of COBE, WMAP and Boomerang) – takes its form

$$H^2 = \frac{1}{3M_{Pl}^2}\rho_\phi. \tag{10}$$

In Eq. (10) ρ_ϕ is the energy density. Assuming that there is only one homogeneous universe (no multiverse) in which the time-dependent events happen the equations of motion are simplified to

$$\frac{d^2\phi_0}{dt^2} + 3H\frac{d\phi_0}{dt} = -\frac{\delta V(\phi_0)}{\delta\phi_0}, \tag{11}$$

$$H^2 = \frac{1}{3M_{Pl}^2}\left(\frac{1}{2}\left(\frac{\partial\phi_0}{\partial t}\right)^2 + V(\phi_0)\right). \tag{12}$$

It seems that the expansion of the inflaton field is successfully described in this model while cooling and later reheating processes are also assumed. It is obvious that the thermodynamics is simply missing. As a first step, this model is developed with the application of the thermal field. For the sake of simplicity, the mass generator Higgs mechanism σ is switched off.

4. LORENTZ INVARIANT DESCRIPTION OF THERMAL PROCESSES

The inflaton field can be coupled with a relativistic theory without contradictions. Thus it is necessary to take the Lorentz invariant formulation of the dissipative processes which was developed earlier [4], [5] independently from the present work. The theory is based on the Lagrange density of a thermal process

$$L_w = \frac{1}{2c^4}\left(\frac{\partial^2\varphi}{\partial t^2}\right)^2 + \frac{1}{2}(\Delta\varphi)^2 - \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2}\Delta\varphi - \frac{1}{2}\frac{c^4c_v^4}{16\lambda^4}\varphi^2, \tag{13}$$

here φ is the scalar generator of the thermal. After the calculation the equation of motion can be obtained

$$\frac{1}{c^4}\frac{\partial^4\varphi}{\partial t^4} + \Delta\Delta\varphi - \frac{2}{c^2}\frac{\partial^2\varphi}{\partial t^2}\Delta\varphi - \frac{c^4c_v^4}{16\lambda^4}\varphi = 0. \tag{14}$$

Here, c is the speed of light in vacuum, c_v is the specific heat at constant volume, and λ is the heat conductivity. Now, the dynamic temperature is defined by

$$T = \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} - \Delta\varphi + \frac{c^2c_v^2}{4\lambda^2}\varphi. \tag{15}$$

Time evolution of the temperature field can be given by the equation

$$\frac{1}{c^2}\frac{\partial^2 T}{\partial t^2} - \Delta T - \frac{c^2 c_v^2}{4\lambda^2} T = 0 , \qquad (16)$$

which is a Klein-Gordon equation with a "negative mass term" (the third one). Moreover, applying the Planck units ($c = 1$; $\hbar = 1$) and introducing the mass of the thermal field

$$M_0 = \frac{\hbar c_v}{2\lambda} , \qquad (17)$$

the Lagrangian can be formulated

$$L_w = \frac{1}{2}\left(\frac{\partial^2 \varphi}{\partial t^2}\right)^2 +$$

$$\frac{1}{2}(\Delta\varphi)^2 - \frac{\partial^2 \varphi}{\partial t^2}\Delta\varphi - \frac{1}{2}M_0^4\varphi^2 . \qquad (18)$$

5. INTERACTION OF THE SCALAR FIELD AND THE THERMAL POTENTIAL FIELD

At this point, the Lagrangian of the interacting fields can be written by not a simply sum of the relevant Lagrangians as follows

$$L_w = \left(\frac{1}{2}\left(\frac{\partial^2 \varphi}{\partial t^2}\right)^2 + \frac{1}{2a^4}(\Delta\varphi)^2 -\right.$$

$$\left.\frac{1}{a^2}\frac{\partial^2 \varphi}{\partial t^2}\Delta\varphi - \frac{1}{2}M_0^4\varphi^2\right) +$$

$$\left(\frac{1}{2}\left(\frac{\partial \phi}{\partial t}\right)^2 - \frac{1}{2a^2}(\nabla\phi)^2 - V(\phi,\varphi)\right), \qquad (19)$$

but taking into account the coupling with the potential term $V(\phi,\varphi)$. There is an optional choice – fitted to the expectations of the theory – for the formulation of the potential of the interacting fields. As a trial calculation it seems to take the least complicated case

$$V(\phi,\varphi) = \frac{1}{2}m^2\phi^2 + \frac{1}{2}g_0^2\phi^2\varphi^2 . \qquad (20)$$

Here, the first term pertains to the inflaton field purely similarly to Eq. (7), the second term expresses the coupling of the two fields. Since the temperature and the dissipation are part of the model through the coupling of the generator potential of the thermal field with the inflaton field, thus the second law of thermodynamics is naturally included in the theory. Applying the method of the calculus of variations the related equations of motion of the homogeneous thermal-inflaton field can be deduced

$$\frac{d^2\phi_0}{dt^2} + 3H\frac{d\phi_0}{dt} = -(m^2 + g_0^2\varphi_0^2)\phi_0 , \qquad (21)$$

$$\frac{d^4\varphi_0}{dt^4} + 6H\frac{d^3\varphi_0}{dt^3} = (M_0^2 + g_0^2\phi_0^2)\varphi_0 , \qquad (22)$$

$$H^2 = \frac{1}{3M_{Pl}^2}\left[\frac{1}{2}\left(\frac{\partial^2\varphi_0}{\partial t^2}\right)^2 -\right.$$

$$\frac{\partial\varphi_0}{\partial t}\frac{\partial^3\varphi_0}{\partial t^3} + \frac{1}{2}\left(\frac{\partial\phi_0}{\partial t}\right)^2 +$$

$$\left.\frac{1}{2}M_0^4\varphi_0^2 + \frac{1}{2}m^2\phi_0^2 + \frac{1}{2}g_0^2\phi_0^2\varphi_0^2\right]. \qquad (23)$$

The dynamic temperature is

$$T = \frac{d^2\varphi_0}{dt^2} + M_0^2\varphi_0 .$$

Those coupled differential equations contain both the dissipative and the non-dissipative energy propagations at the same time [4], [6], [24], [25] stabilizing the solution rolling towards the thermal equilibrium. Furthermore, it is generally accepted that the dissipative processes must have been an indispensable role in the evolution of the early universe [26], [27].

6. RESULTS AND DISCUSSIONS

In the present paper, the coupling of the inflaton field ϕ and the generator field of dissipative – non-dissipative energy propagation φ is treated to widen the Linde's inflationary model of the early universe. The above equations are solved numerically with the parameter set: $m = 80$ GeV suggested by Linde, $M_0 = 52.2$ GeV, and $g_0 = 0.12$ GeV. It is immediately emphasized that further calculations are needed to map the physically relevant solutions. The results show that in the energy density dominated inflation regime the inflaton field ϕ decays (Fig. 1) into the thermal field (Fig. 2). In spite of this energy conversion, the thermal field φ is decreasing since the cooling expansion is much dominated. Then a reheating process appears in a post-inflationary process. This situation arises during the oscillating period of the inflaton field (Fig. 1) while the thermal field (Fig. 2) and the temperature (Fig. 3) increase due to the inflaton-thermal field energy transfer. The normalized radius of the universe (Fig. 4), the energy density (Fig. 5) and the pressure (Fig. 6) are also calculated and plotted.

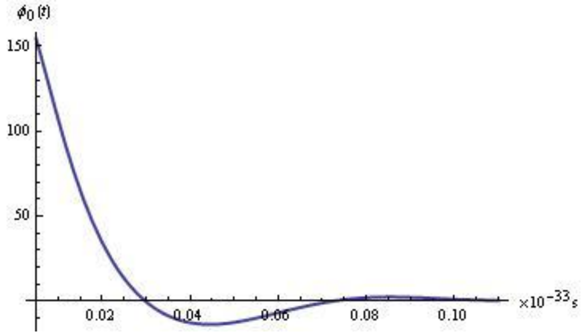

Figure 1.
Time-dependence of the inflaton field shows its decay and the starting oscillation

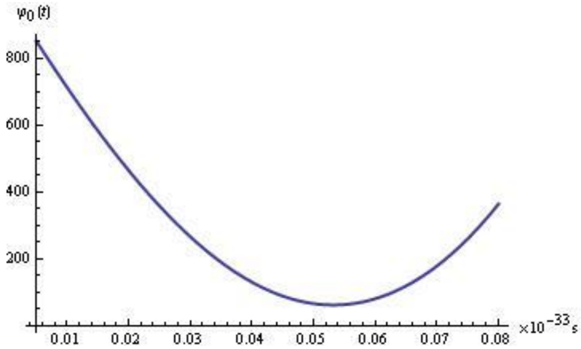

Figure 2.
Time-dependence of the thermal field

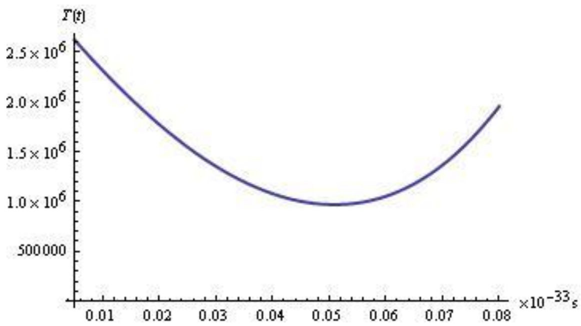

Figure 3.
Time-dependence of the temperature field

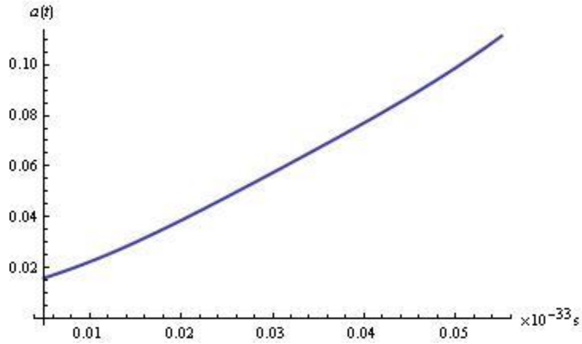

Figure 4.
The normalized radius of the expanding universe as a function of time

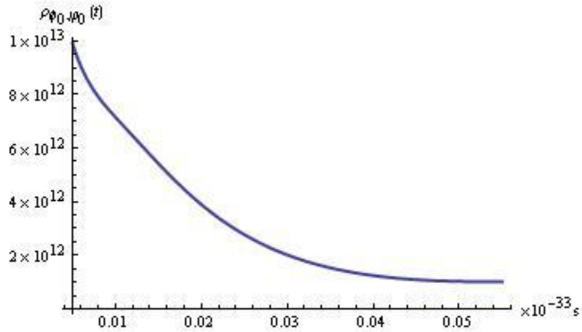

Figure 5.
Decreasing energy density of the universe in time

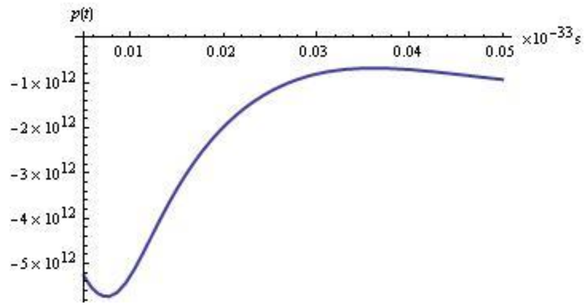

Figure 6.
Time-dependence of the pressure

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