CHANGE OF THE DYNAMICS OF THE SYSTEMS: DISSIPATIVE – NON-DISSIPATIVE TRANSITION

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SUMMARY

Dynamic phase transitions between non-dissipative and dissipative processes are discussed from different viewpoints. Mechanical examples are shown to illustrate the transition pointing out their realistic behavior. The phase transition is shown on a "stretched string on a rotating wheel" system. In the thermal energy transport an abstract scalar field has been introduced to generate a dynamical temperature and a covariant field equation to describe the heat propagation with finite speed—less than the speed of light—of action. It has been shown how this scalar field can be connected to the usual temperature (local equilibrium temperature) and the Fourier's heat conduction. Mathematically, Klein–Gordon equations with a "negative" mass term describe this spinodal instability. The dynamical phase transition is in between these two kinds of – wave and non-wave – propagation, or with an other context, it is better to say, a dynamical phase transition between a non-dissipative and a dissipative thermal process. It seems interesting that the thermal case may have an important role in the definition of a really dynamical temperature.

MECHANICAL EXAMPLES

In the model we take a stretched string with fixed ends subject an additional force density perpendicular to the axis of the string as it is shown in Fig. 1. We can construct the Lagrangian L(-), Eq. (1 with -), then the equation of motion as the Euler–Lagrange equation Eq. (2 with +) can be obtained

$$L(\mp) = \int \left[\begin{array}{l} \frac{1}{2}\rho A \left(\frac{\partial \Psi}{\partial t} \right)^2 - \\ -\frac{1}{2} F \left(\frac{\partial \Psi}{\partial x} \right)^2 \mp \frac{1}{2} D \Psi^2 \end{array} \right] dx, \qquad (1)$$

$$\frac{\partial^2 \Psi}{\partial t^2} - \frac{F}{\rho A}\frac{\partial^2 \Psi}{\partial x^2} \pm \frac{D}{\rho A}\Psi = 0, \qquad (2)$$

where Ψ is the normal displacement, ρ is the mass density, A is the cross section of the string, F is the stretching force and D is the spring force density. This is the well-known classical form of the Klein–Gordon equation. If we assume the existence of repulsive-like springs changing the sign of the potential, we get Fig. 2. Applying this repulsive potential we obtain the Lagrangian L(+), Eq. (1 with +) and the equation of motion Eq. (2 with -). This equation can be also handled as a Klein–Gordon equation, however its solution is quite different from the previous case [1]. It has a so-called "tachyonic"-like solution [2] that brings the possibility of a dynamical phase transition [3] into the theory. If we put the stretched string on a rotating disk when the origin of the disk is on the line (and not at the end point, practically in the middle) shown in Fig. 3,

$$\frac{\partial^2 \Psi}{\partial t^2} - \frac{F}{\rho A}\frac{\partial^2 \Psi}{\partial x^2} - \omega^2 \Psi = 0 \qquad (3)$$

the appearing centrifugal force in Eq. (3) behaves, as a repulsive interaction from this viewpoint (ω is the angular velocity). This potential has the same structure like the spring force in Eq. (2). We calculate the dispersion relation by Eq. (4), the phase velocity by Eq. (5), the group velocity by Eq. (6).

$$\Omega(k,\omega) = \sqrt{\frac{F}{\rho A}k^2 - \omega^2}\,, \qquad (4)$$

$$w_{ph} = \frac{\Omega}{k} = \sqrt{\frac{F}{\rho A} - \left(\frac{\omega}{k}\right)^2} \qquad (5)$$

and

$$v_g = \frac{d\Omega}{dk} = \frac{1}{\sqrt{1 - \frac{\rho A}{F}\left(\frac{\omega}{k}\right)^2}}\sqrt{\frac{F}{\rho A}}. \qquad (6)$$

Examining these equations we can see from Eq. (5) that there is a critical value of angular velocity ω, which is the upper limit of the wave propagation. The physical meaning is clear, above this value the centrifugal force elongates the string to infinity, i.e., the string cannot have vibrating modes. Summarizing, we can say that wave modes exist if Eq. (7a) is valid and there are no wave modes if Eq. (7b) relation is realized. The dynamical phase transition happens at a certain value of k as it can be seen in Fig. 4

$$\frac{F}{\rho A} > \left(\frac{\omega}{k}\right)^2 \quad (7a), \qquad \frac{F}{\rho A} < \left(\frac{\omega}{k}\right)^2. \quad (7b)$$

DISSIPATIVE – NON-DISSIPATIVE TRANSITION

We have introduced an abstract scalar field φ to generate a dynamical temperature T and a covariant field in Eq. (8) to describe the heat propagation with finite speed—less than the speed of light—of action

$$T(x,t) = \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} - \frac{\partial^2\varphi}{\partial x^2} + \frac{c^2 c_v^2}{4\lambda^2}\varphi. \quad (8)$$

Temperature τ denotes the usual local equilibrium temperature given by

$$\tau = -\frac{\partial\varphi}{\partial t} - \frac{\lambda}{c_v}\frac{\partial^2\varphi}{\partial x^2}. \quad (9)$$

The relevant equations parallel both the Lagrangian L_w for the wave solution and Lagrangian L_c for the classical heat conduction [4-8]

$$L_w = \frac{1}{2}\left(\frac{\partial^2\varphi}{\partial x^2}\right)^2 + \frac{1}{2c^4}\left(\frac{\partial^2\varphi}{\partial t^2}\right)^2 - \\ -\frac{1}{c^2}\frac{\partial^2\varphi}{\partial x^2}\frac{\partial^2\varphi}{\partial t^2} - \frac{1}{2}\frac{c^4 c_v^4}{16\lambda^4}\varphi^2, \quad (10)$$

$$L_c = \frac{1}{2}\left(\frac{\partial\varphi}{\partial t}\right)^2 + \frac{1}{2}\frac{\lambda^2}{c_v^2}\left(\frac{\partial^2\varphi}{\partial x^2}\right)^2. \quad (11)$$

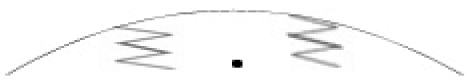

Figure 1.
Stretched string with attractive potential

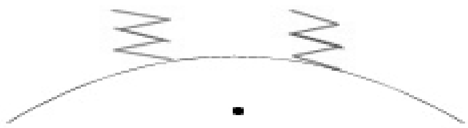

Figure 2.
Stretched string with repulsive potential

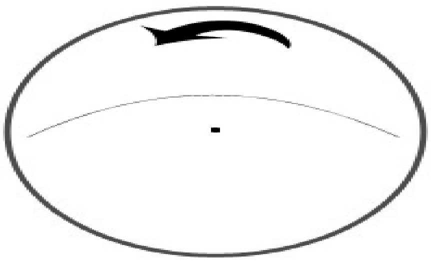

Figure 3.
Stretched string on a rotating disc

Eqs. (12) and (13) show the difference between the propagations. The Klein–Gordon type Eq. (12) with a repulsive potential can serve a tachyonic solution leading to the spinodal instability [9]. The parabolic Eq. (13) pertains to the Fourier's heat equation. We calculate the dispersion relations for both cases from which we obtain the phase and group velocities. If we tend to the infinity with speed of light then the group velocity of Eq. (12) tends to the group velocity of Eq. (13)

$$\frac{1}{c^2}\frac{\partial^2 T}{\partial t^2}-\frac{\partial^2 T}{\partial x^2}-\frac{c^2 c_v^2}{4\lambda^2}T=0, \qquad (12)$$

$$\frac{\partial \tau}{\partial t}-\frac{\lambda}{c_v}\frac{\partial^2 \tau}{\partial x^2}=0. \qquad (13)$$

The dispersion relations for these equations are

$$\omega(k)=\sqrt{c^2k^2-\frac{c^4c_v^2}{4\lambda^2}} \qquad (14a)$$

and

$$\omega(k)=-i\frac{\lambda}{c_v}k^2. \qquad (14b)$$

The group velocity v_g for the Klein–Gordon equation given by Eq. (14a) is

$$v_g=\frac{d\omega}{dk}=\frac{c}{\sqrt{1-\frac{c^2}{4D^2k^2}}}. \qquad (15)$$

If the speed of light tends to infinite this relation turns into $-i2Dk$ which is exactly the group velocity of the classical equation. Similarly, we can calculate the phase velocity w_{ph} for the wave equation

$$w_{ph}=\frac{\omega}{k}=c\sqrt{1-\frac{c^2}{4D^2k^2}}<c. \qquad (16)$$

Now, let $k_0=\frac{c}{2D}$ be, and it can be seen that if $k>k_0$ then $\frac{d\omega}{dk}$ is a real and wave solution. However, if $k<k_0$ then $\frac{d\omega}{dk}$ is imaginary and non-wave. If we plot w_{ph} and $\mathrm{Im}\, w_{ph}$ as a function of Dk we can realize that close to Dk_0 the critical slow down appears in Fig 4.

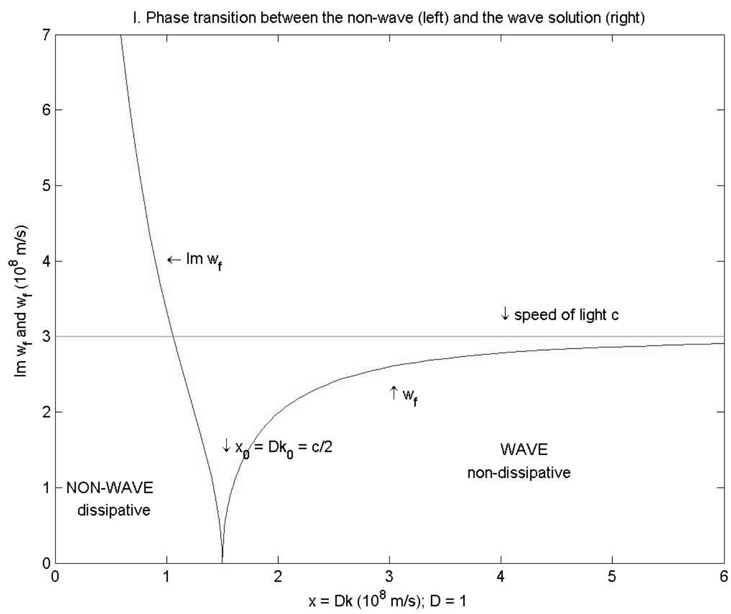

Figure 4.
Phase transition diagram

ACKNOWLEDGMENT

The author would like to thank the National Office of Research and Technology (NKTH; Hungary) for financial support MX-20/2007 (Grant No. OMFB-00960/2008).

REFERENCES

[1] Gambár K., Márkus F. *A simple mechanical model to demnstrate a dynamical phase transition.* Rep. Math. Phys. **62**, 219 (2008).
[2] Bollini C.G., Rocca M.C., *The Wheeler propagator,* Int. J. Theor. Phys. **37**, 2877 (1998).
[3] Shang-keng Ma. *Modern Theory of Critical Phenomena.* (Addison-Wesley, California, 1982).
[4] Gambár K. *Least action principle for dissipative processes.* (in Variational and extremum principles in macroscopic systems, Eds. S. Sieniutycz and H. Farkas) (Elsevier, Amsterdam, 2005); pp. 245-266.
[5] Gambár K., Márkus F. *Hamilton-Lagrange formalism of nonequilibrium thermodynamics.* Physical Review E **50**, 1227 (1994).
[6] Gambár K., Martinás K., Márkus F. *Examination of phenomenological coefficient matrices within the canonical model of field theory of thermodynamics.* Physical Review E **55**, 5581 (1997).
[7] Gambár K., Márkus F. *A possible dynamical phase transition between the dissipative and the non-dissipative solutions of a thermal process.* in press, Phys. Lett. A **361,** 283 (2007).
[8] Gambár K. *A dynamical phase transition between the dissipative and the non-dissipative solutions of a thermal process and its consequences.* (IWNET 2009 Conference, Cuernavaca, Mexico, August 24-28, 2009).
[9] Polonyi J. *Spinodal Instability and Confinement.* hep-th/0509078 (Talk presented at the Workshop GRIBOV-75, Budapest, May 2005).