APPLICATION OF TAGUCHI-QUALITY ASSURANCE METHOD IN CASE OF MECHANICAL STRUCTURES, ON THE BASIS OF SSI MODEL

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SUMMARY

A basic principle of the Taguchi method is the decrease of the variability of parameters. While constructing and manufacturing mechanical structures this means the decrease of the dispersion of strength. During operation, stress is fluctuating as well. At the stipulated level of reliability the acceptable dispersion of stress may be determined by means of the SSI model, while the acceptable dispersion of strength may be determined knowing the fluctuation of stress. The model may be efficient not only for mechanical structures but also for other types of mechanical equipment and even for modelling social conflicts as well.

PRINCIPLES OF TAGOUCHI METHOD (TM)

The variety of classical quality control methods is very wide eg. that of Genichi Taguchi [i1], [i2] (Japanese developer engineer) is widely used even today. He was working on development of Japan telephone network just after WW2. He's method is summarized in seven principles:

- The loss caused to the society is an important characteristic of the product.
- The loss of the consumer is proportional to the square of deviation related to purpose to be achieved.
- Permanent product development is required in the competition.
- The quality and cost of the product is mainly dominated by the designer.
- The variability of operation should be reduced.
- This latter object may be obtained by statistical experiment.
- It may be obtained by exploitation of nonlinear phenomena too.

Now we focus on the fifth principle, which may be the most important from engineering point of view.

Table 1.
Fundamentals of TM

Control factors,	*Noise factors*	Operational parameter,
which may be influenced by us	TM does not try to reduce them, but to support their effects	Characterizes the quality of the product or the process

While optimizing, Taguchi classifies control factors into two groups. Those in the first group are fixed after static optimizing, while those in the second group are subject to permanent variation during the operation. This latter is known as fine adjusting. During the planning and the operation process the designer controls the variability of either component; however a theory is required which creates relation between structural and operational parameters, while a predefined operational probability should be obtained. It is evident that a product fails when the current value of its charge higher than its capacity.

THEORY OF THE SSI-MODEL AND ITS CALCULATION

Previously the reasons of failure were dealt in a schematic way. Now the probability of occurrence will be determined by a mathematical way. There are several methods to determine the failure-probability of technical systems due to a single charge. One of these methods is the charge-capacity overlap method, well known in the international references [1], [2] as Stress-Strength Interference Technique (SSI).

When several factors are influencing the capacity of a mechanical structure (e.g. size variation of a critical structural element, variation of alloy quantities, variation of heat treatment time and temperature), on the basis of central limit-distribution principle it is supposed that the capacity (e.g. expressed in mechanical stress) is a probability variable with normal distribution and with an expected value of m_t and σ_t variation.

The charge is the result of several effects, as the parameters may be anywhere within the size and position tolerance of the components, while the active charge may vary operationally, consequently when the variable is considered as mechanical stress, it may be modelized by means of variable of a standard distribution, the parameters of which are m_i and σ_i In case of traditional designing process the difference between the expected values are predicted by means of an industrial standard or by any other designing guideline taking into consideration a proper coefficient of security. Due to the variation it may happen that the signum of the variable's difference will change related to the expected one. This phenomenon is illustrated on figure 1. where the black-crosshatched area is proportional with the probability of the failure.

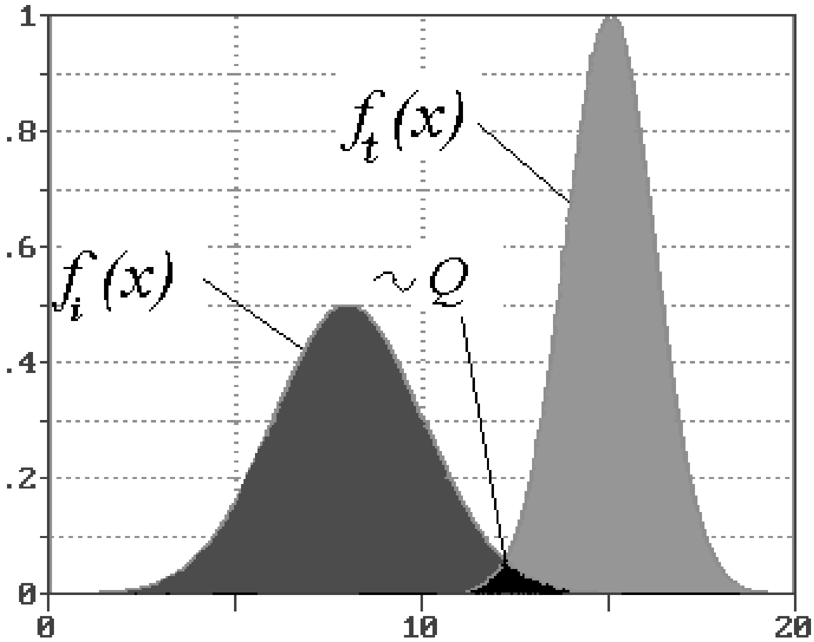

Figure 1.
Charge versus capacity distribution

It may be proved that the probability of failure is:

$$Q = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-SM} e^{-\frac{u^2}{2}} du = \Phi(-SM) = 1 - \Phi(SM)$$

where SM (safety margin)

$$SM = \frac{m_t - m_i}{\sqrt{\sigma_t^2 + \sigma_i^2}}$$

SM is a variable characterizing the security of operation, which may be calculated when the distributions are known. Values of $\Phi(x)$ are available. The probability of failure-less operation is:

$$R = 1 - Q = \Phi(SM)$$

Prove:

The characteristic function of the normal distribution is ([3] 267. page.):

$$\varphi(t) = e^{itm - \sigma^2 \frac{t^2}{2}}$$

When ξ and η are independent probability variables according to $N(m_t, \sigma_t)$ and $N(m_i, \sigma_i)$ distribution, on the basis of series theory ([3] 265. page) related to sum and difference, the characteristic function of the difference, say $\zeta = \xi - \eta$ is as follows:

$$\varphi_{\text{difference}}(t) = e^{itm_t - \sigma_t^2 \frac{t^2}{2}} e^{-itm_i - \sigma_i^2 \frac{t^2}{2}} =$$

$$= e^{it(m_t - m_i) - (\sigma_t^2 + \sigma_i^2)\frac{t^2}{2}},$$

From the above equation it is concluded that ζ shows a normal distribution as well:

$$N\left(m_t - m_i, \sqrt{\sigma_t^2 + \sigma_i^2}\right)$$

The distribution function is:

$$P(\varsigma < z) = F(z) =$$

$$= \frac{1}{\sqrt{2\pi(\sigma_t^2 + \sigma_i^2)}} \int_{-\infty}^{z} e^{-\frac{(x - m_t + m_i)^2}{2(\sigma_t^2 + \sigma_i^2)}} dx =$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{z - m_t + m_i}{\sqrt{\sigma_t^2 + \sigma_i^2}}} e^{-\frac{u^2}{2}} du, \quad \text{ahol} \quad u = \frac{x - m_t + m_i}{\sqrt{\sigma_t^2 + \sigma_i^2}}$$

The failure occurs when the difference becomes a negative value. The probability of this event is given by the reading from distribution function, when $z=0$.

$$Q = P(\varsigma < 0) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{m_i - m_t}{\sqrt{\sigma_t^2 + \sigma_i^2}}} e^{-\frac{u^2}{2}} du =$$

$$= \Phi(-SM) = 1 - \Phi(SM) \Rightarrow$$

$$\Rightarrow R = P(\varsigma \geq 0) = \Phi(SM)$$

APPLICATIONS

A) The nominal capacity and the charge of a mechanical equipment produced in series shows normal distribution. All parameters excepted one is known. The capacity is $m_t=500$ *MPa* $\sigma_t=50$ *MPa*, the charge is $m_i=300$ *MPa*, $\sigma_i=?$ should be determined. It should be determined what a variation of charge is acceptable when the variation of operation is known and $R=0{,}9900$ is the required probability of the failure-less operation.

Calculations

The following data are used from the standard distribution's function. The details of the complete table are in Table 2.

Let we determine the value of SM by means of linear interpolation. The mathematical steps are: replacement of data into the inverse function, so it is an ulterior recall of values.

$$R = \Phi(SM)$$

$$SM \approx 2{,}32 + \frac{0{,}9900 - 0{,}9898}{0{,}9904 - 09898} \cdot 0{,}02 =$$

$$= 2{,}32 + \frac{2 \cdot 0{,}02}{6} \approx 2{,}3267$$

$$SM = \frac{m_t - m_i}{\sqrt{\sigma_t^2 + \sigma_i^2}} = \frac{500 - 300}{\sqrt{50^2 + \sigma_i^2}}$$

$$\Rightarrow \sigma_i^2 = \left(\frac{200}{2{,}3267}\right)^2 - 2500 = 4888{,}89\left[\text{MPa}^2\right]$$

$$\sigma_i = \sqrt{4888{,}89} = 69{,}92 \approx 70[\text{MPa}]$$

Consequently the given equipment should be operated that the variation of the expected charge of *300 MPa* must not exceed *70 MPa*.

B) Taking into consideration the two expected values above, the coefficient of security is:

$$n_b = \frac{500}{300} = 1{,}\dot{6}$$

What will be the security coefficient with the same value of R, if the charge remained a probability variable with $N(300{,}70)$ distribution, but without variation in capacity?

Table 2.

x	2,30	2,32	2,34	2,36
$\Phi(x)$	0,9893	0,9898	0,9904	0,9909

Calculations

$$m_{t\min} = m_i + SM\sigma_i = 300 + 2{,}3267 \cdot 70 =$$
$$= 462{,}869 \approx 463[\mathrm{MPa}]$$
$$n_{b\min} = \frac{463}{300} = 1{,}54\dot{3}$$

C) In the practice it is impossible to arrive to a variation-less charge, which case would yield theoretically to the use of cheapest structural material. What will be the coefficient of security and the expected value with constant R if the variation of the capacity would be reduced to the half of the original one, say *25 MPa*?

Calculations

$$m_t = m_i + SM\sqrt{\sigma_t^2 + \sigma_i^2} =$$
$$= 300 + 2{,}3267\sqrt{25^2 + 70^2} =$$
$$= 472{,}94441 \approx 473[\mathrm{MPa}]$$
$$n_{b\mathrm{real}} = \frac{473}{300} = 1{,}57\dot{6}$$

OTHER APPLICATIONS OF THE SSI MODEL

As we've seen in the introduction, the SSI model is applicable not only in case of mechanical structures. In case of electrical devices the charge and the capacity may be expressed in current flux, as the current could serve as analogy for the force or for the torque. In case of contra-flood dams SM is widely used instead of security coefficient [4].

Risk prediction of specialists and danger-feeling of laymen shows separate variation and the average value of them may show a considerable difference. In aiming to deal with conflicts arising from this phenomenon it is suggested a probability method analog with the SSI [5].

REFERENCES

[1] R. L. Disney, C. Lipson and N. J. Sheth. *The determination of the Probability of Failure by Stress/Strength Interference Theory.* IEEE Symposium on Reliability, Jan. 1968. in: Dr. Petrik Olivér. *Berendezések megbízhatóságra tervezésének néhány kérdése.* Finommechanika-Mikrotechnika, 28. évf. 1989. március, pp. 65-72

[2] J. Davidson. *The reliability of mechanical systems.* ImechE Guides for the Process Industries, Mechanical Engineering Publications Limited for Institution of Mechanical Engineers London, 1968 in: Dr. Gaál Zoltán, Dr. Kovács András. *Megbízhatóság, karbantartás.* 1994. Veszprémi Egyetem, Kiadói Iroda p. 342

[3] Prékopa András. *Valószínűségelmélet műszaki alkalmazásokkal.* Műszaki Könyvkiadó, Bp., 1962. p. 440

[4] Nagy László. *Az árvízi biztonság fejlődése.* Hidrológiai Közlöny, 80. évf. 2. szám, 2000. pp. 114-115

[5] Dr. Benedikt Szvetlána, Dr. Kun István, Dr. Szász Gábor. *SSI modellel analóg kockázatelemzés.* INFORMATIKA, IX. évf. 2. szám. 2007. május, pp. 33-36.

Internet:

[6] [i1] Taguchi, Genichi (June 1995). "Quality engineering (Taguchi methods) for the development of electronic circuit technology". *IEEE Transactions on Reliability* (IEEE Reliability Society) **44** (2): *225-229.* doi:10.1109/24.387375. ISSN 0018-9529. http://ieeexplore.ieee.org/xpl/freeabs_all.jsp?tp=&arnumber=387375&isnumber=8781.

[7] [i2] http://www.amsup.com/bios/g_taguchi.html